Lecture 17 - March 18

Binary Search, Merge Sort

Binary Search: Tracing, Running Time
MergeSort: Ideas, Java, Tracing

Announcements/Reminders

- Assignment 3 (on linked Trees) solution released
- WrittenTest and ProgTest1 results & feedback released
- ProgTest2 guide & example questions to be released
- Makeup Lecture (on Queues) posted
- Lecture notes template, Office Hours, TA Contact

Binary Search: Tracing

a →

0	1	2	3	4	5	6	7	8
3	6	9	12	15	18	21	24	27

→ initial search space.

search(a,18) key

binarySearchH(a,0,8,18)

$m = \frac{0+8}{2} = \boxed{4}$

m.v. a[4] == 15

18 > 15

↳ go R.

(4+1, 8)

binarySearchH(a,5,8,18)

$m = \frac{5+8}{2} = 6$

m.v. a[6] == 21

18 < 21

↳ go L.

↳ (5, 6-1)

binarySearchH(a,$\boxed{5,5}$,18)

a[5] == 18

true → found it!

---

a →

0	1	2	3	4	5	6	7	8
3	6	9	12	15	18	21	24	27

Exercise.

search(a,7)

binarySearchH(a,0,8,7)

binarySearchH(a,0,3,7)

binarySearchH(a,2,3,7)

binarySearchH(a,2,1,7)

Binary Search: Running Time

```java
boolean binarySearch(int[] sorted, int key) {
  return binarySearchH(sorted, 0, sorted.length - 1, key);
}
boolean binarySearchH(int[] sorted, int from, int to, int key) {
  if (from > to) { /* base case 1: empty range */
    return false; }
  else if(from == to) { /* base case 2: range of one element */
    return sorted[from] == key; }
  else {
    int middle = (from + to) / 2;
    int middleValue = sorted[middle];
    if(key < middleValue) {
      return binarySearchH(sorted, from, middle - 1, key);
    }
    else if (key > middleValue) {
      return binarySearchH(sorted, middle + 1, to, key);
    }
    else { return true; }
  }
}
```

n

$T(n) = ?$

$T(0) = 1$

$T(1) = 1$

$O(1)$

$T(n)$

$n/2$

assume: $T(\frac{n}{2})$

assume: $T(\frac{n}{2})$

$n/2$

Running Time as a Recurrence Relation

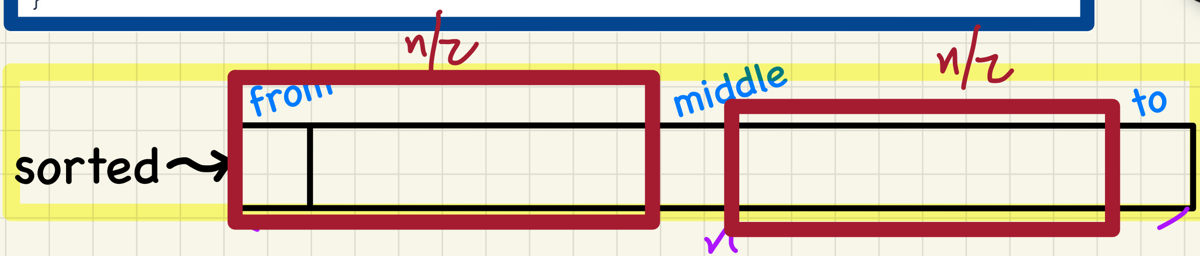
$T(0) = 1$

$T(1) = 1$

$T(n) = \boxed{T(\frac{n}{2}) + 1}$

wrong: (∵ if-elseif)

$T(n) = T(\frac{n}{2}) + T(\frac{n}{2}) + 1$



Running Time: Unfolding Recurrence Relation

$T(0) = 1$

$T(1) = 1$

$T(n) \overset{\checkmark}{=} T(n/2) + 1$

Assume without loss of generality:

$$n = 2^x \text{ for } x \geq 0$$

Warm-up: $T(\frac{n}{2}) = T(\frac{n/2}{2}) + 1 = T(\frac{n}{4}) + 1$

$$T(n) = T(\tfrac{n}{2}) + 1 \qquad 2^1$$

$$= \left(T(\tfrac{n}{4}) + 1 \right) + 1 \qquad 2^2$$

$$= \left(\left(T(\tfrac{n}{8}) + 1 \right) + 1 \right) + 1 \qquad 2^3$$

$$\vdots$$

how many? (hm) $\log n$

$$1 \quad \boxed{T(1)} \; \boxed{+1 + 1 \cdots + 1} = 1 + \log n \cdot 1 = \log n$$

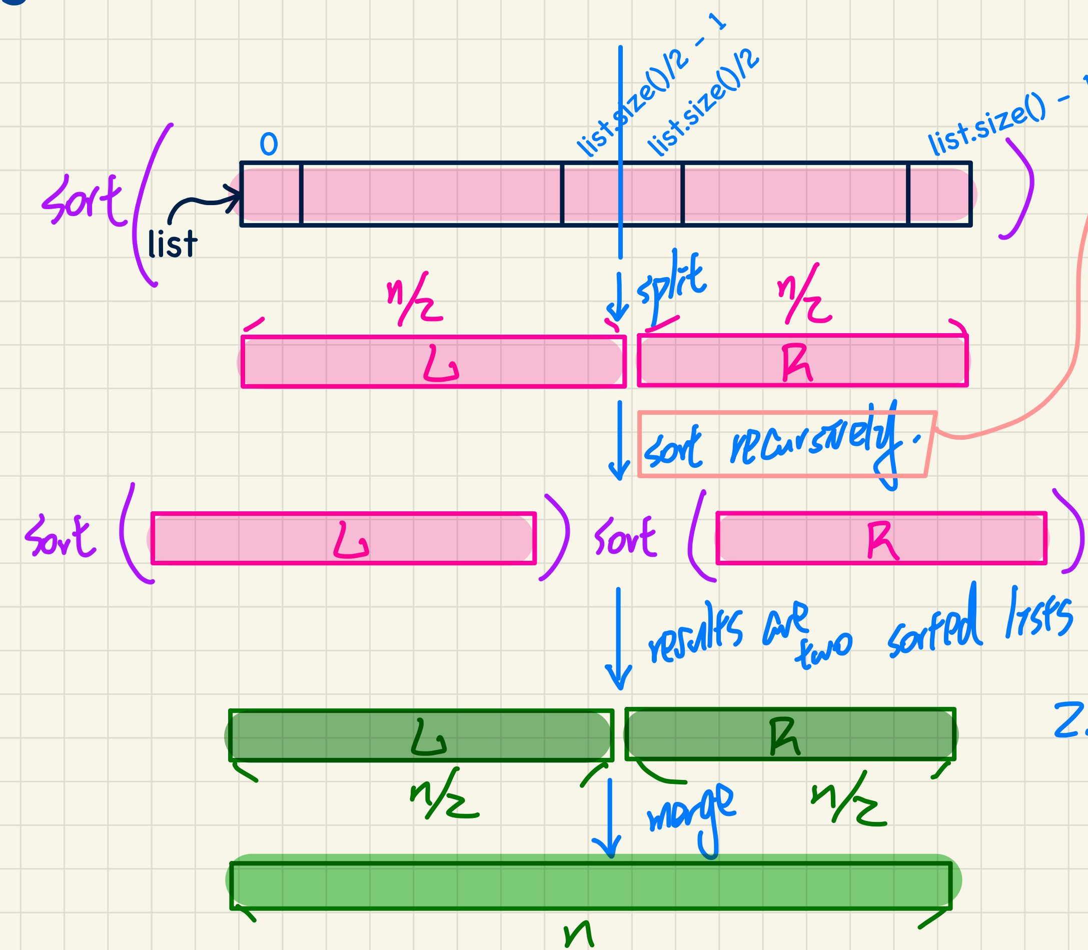
$$1 = \frac{n}{n} = \frac{n}{2^{hm}} = \frac{n}{2^{\log n}}$$

$n = 8$

$$\frac{8}{2^{\log 8}} = 1 \qquad 3$$

Merge Sort: Ideas

- split
- merge



Each recursive call:

1. $\boxed{\text{split}}$ ($n \to \frac{n}{2}, \frac{n}{2}$,
$\frac{n}{2} \to \frac{n}{4}, \frac{n}{4}$,
$\frac{n}{4} \to \frac{n}{8}, \frac{n}{8}$,
⋮
$2 \to \boxed{1}, \boxed{1}$)

$O(1)$

sorted

$O(n)$

2. $\boxed{\text{Merge}}$ ($1, 1 \to 2$,
$2, 2 \to 4$,
⋮
$\frac{n}{2}, \frac{n}{2} \to n$)

Merge Sort in Java

Total # iterations for merge: (L.size + j) + (R.size − j) = L.size + R.size

* Exhaust either L or R.

e.g. L is exhausted.

≈ i + j iterations so far (i ≈ L.size)

** Append remaining ele. from R.

≈ R.size − j iterations

base cases

```java
public List<Integer> sort(List<Integer> list) {
  List<Integer> sortedList;
  if(list.size() == 0) { sortedList = new ArrayList<>(); }
  else if(list.size() == 1) {
    sortedList = new ArrayList<>();
    sortedList.add(list.get(0));
  }
  else {
    int middle = list.size() / 2;
    List<Integer> left = list.subList(0, middle);
    List<Integer> right = list.subList(middle, list.size());
    List<Integer> sortedLeft = sort(left);
    List<Integer> sortedRight = sort(right);
    sortedList = merge(sortedLeft, sortedRight);
  }
  return sortedList;
}
```

O(L.size + R.right)

i

left

0	1	2	3
24	45	63	85

j

right

0	1	2	3
17	31	50	96

L is exhausted

Precondition

L and R sorted.

merge

17	24	31	45	50	63	85	96

```java
/* Assumption:  L and R are both already sorted. */
private List<Integer> merge(List<Integer> L, List<Integer> R) {
  List<Integer> merge = new ArrayList<>();
  if(L.isEmpty() || R.isEmpty()) { merge.addAll(L); merge.addAll(R); }
  else {
    int i = 0;
    int j = 0;
    while(i < L.size() && j < R.size()) {
      if(L.get(i) <= R.get(j)) { merge.add(L.get(i)); i ++; }
      else { merge.add(R.get(j)); j ++; }
    }
    /* If i >= L.size(), then this for loop is skipped. */
    for(int k = i; k < L.size(); k ++) { merge.add(L.get(k)); }
    /* If j >= R.size(), then this for loop is skipped. */
    for(int k = j; k < R.size(); k ++) { merge.add(R.get(k)); }
  }
  return merge;
}
```

exit: i >= L.size || j >= R.size

* (while loop) O(1)

** (for loops) O(1)

append the remaining elements from the unexhausted list.

Merge Sort: Tracing

split

merge